NAME ______________________ DATE __________ PERIOD _____

7-8 Study Guide and Intervention

Inverse Functions and Relations

Find Inverses - Switch x and y

Inverse Relations	Two relations are inverse relations if and only if whenever one relation contains the element (a, b), the other relation contains the element (b, a).
Property of Inverse Functions	Suppose f and f^{-1} are inverse functions. Then $f(a) = b$ if and only if $f^{-1}(b) = a$.

Ex: $f = \{(3,7),(2,-5),(-4,0)\}$

$f^{-1} = \{(7,3),(-5,2),(0,-4)\}$

Example **Find the inverse of the function $f(x) = \frac{2}{5}x - \frac{1}{5}$. Then graph the function and its inverse.**

Step 1 Replace $f(x)$ with y in the original equation.

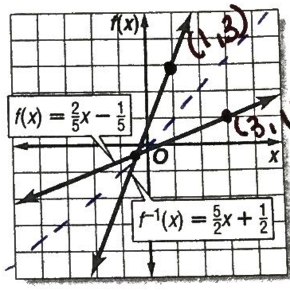
$$f(x) = \frac{2}{5}x - \frac{1}{5} \rightarrow y = \frac{2}{5}x - \frac{1}{5}$$

Step 2 Interchange x and y.

$$x = \frac{2}{5}y - \frac{1}{5}$$

Step 3 Solve for y.

$5\left(x = \frac{2}{5}y - \frac{1}{5}\right)$	Inverse
$5x = 2y - 1$	Multiply each side by 5.
$5x + 1 = 2y$	Add 1 to each side.
$\frac{1}{2}(5x + 1) = y$	Divide each side by 2.



Graphs = reflected over $y=x$

The inverse of $f(x) = \frac{2}{5}x - \frac{1}{5}$ is $f^{-1}(x) = \frac{1}{2}(5x + 1)$.

$= \frac{5}{2}x + \frac{1}{2} = \frac{5x+1}{2}$

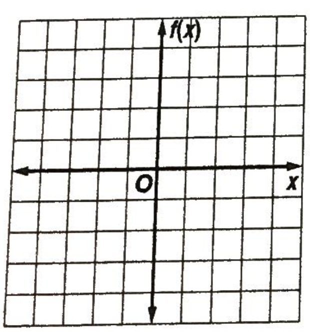
Exercises

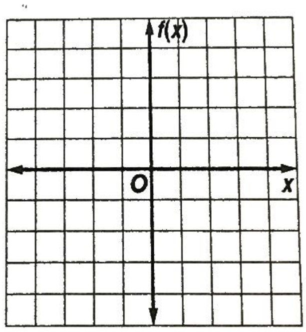
Find the inverse of each function. Then graph the function and its inverse.

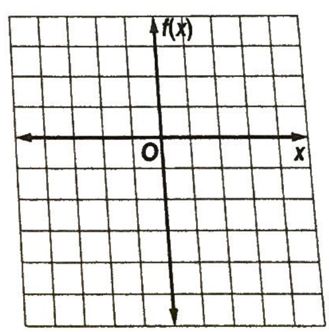
1. $f(x) = \frac{2}{3}x - 1$

2. $f(x) = 2x - 3$

3. $f(x) = \frac{1}{4}x - 2$







Lesson 7-8

7-8 Study Guide and Intervention *(continued)*

Inverse Functions and Relations

Inverses of Relations and Functions

Inverse Functions	Two functions f and g are inverse functions if and only if $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Example 1 **Determine whether $f(x) = 2x - 7$ and $g(x) = \frac{1}{2}(x + 7)$ are inverse functions.**

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left[\frac{1}{2}(x + 7)\right] \\ &= 2\left[\frac{1}{2}(x + 7)\right] - 7 \\ &= x + 7 - 7 \\ &= x\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g(2x - 7) \\ &= \frac{1}{2}(2x - 7 + 7) \\ &= x\end{aligned}$$

The functions are inverses since both $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Example 2 **Determine whether $f(x) = 4x + \frac{1}{3}$ and $g(x) = \frac{1}{4}x - 3$ are inverse functions.**

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f\left(\frac{1}{4}x - 3\right) \\ &= 4\left(\frac{1}{4}x - 3\right) + \frac{1}{3} \\ &= x - 12 + \frac{1}{3} \\ &= x - 11\frac{2}{3}\end{aligned}$$

Since $[f \circ g](x) \neq x$, the functions are not inverses.

Exercises

Determine whether each pair of functions are inverse functions.

1. $f(x) = 3x - 1$
$g(x) = \frac{1}{3}x + \frac{1}{3}$

2. $f(x) = \frac{1}{4}x + 5$
$g(x) = 4x - 20$

3. $f(x) = \frac{1}{2}x - 10$
$g(x) = 2x + \frac{1}{10}$

4. $f(x) = 2x + 5$
$g(x) = 5x + 2$

5. $f(x) = 8x - 12$
$g(x) = \frac{1}{8}x + 12$

6. $f(x) = -2x + 3$
$g(x) = -\frac{1}{2}x + \frac{3}{2}$

7. $f(x) = 4x - \frac{1}{2}$
$g(x) = \frac{1}{4}x + \frac{1}{8}$

8. $f(x) = 2x - \frac{3}{5}$
$g(x) = \frac{1}{10}(5x + 3)$

9. $f(x) = 4x + \frac{1}{2}$
$g(x) = \frac{1}{2}x - \frac{3}{2}$

10. $f(x) = 10 - \frac{x}{2}$
$g(x) = 20 - 2x$

11. $f(x) = 4x - \frac{4}{5}$
$g(x) = \frac{x}{4} + \frac{1}{5}$

12. $f(x) = 9 + \frac{3}{2}x$
$g(x) = \frac{2}{3}x - 6$